AENG 411: Aerospace Laboratory

Vibration of a Cantilever Beam

&

Resonance Response of a Cantilever Wing

by

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# Summary

The objective of this experiment was to utilize the wind tunnel in Oliver Hall to analyze the flow characteristics associated with a model aircraft. This was accomplished by attaching the model to a tunnel balance that is capable of measuring forces and moments in 6 separate directions. Before the data collected from this balance could be used, however, it was necessary to account for disruptions in the flow around the model that developed as a result of flow interactions between the walls of the tunnel and the freestream flow, as well as the presence of the aircraft itself. Once these errors were accounted for, the data that resulted was analyzed, as to determine the stability characteristics associated with the aircraft. Ultimately, it was found that the data collected was highly impacted by error associated with the use of the tunnel balance, even when the error corrections were taken into account. Even with these errors, it was still possible to determine the lift curve slope, minimum drag, maximum lift, and static margin associated with the aircraft. All of these results can be found in Sections (7-8), with the raw data being stored in Section (9). Thus, the lab as a whole was a success, in that the flow characteristics associated with a model aircraft were determined through the use of a wind tunnel.

# Nomenclature

*I* Area Moment of Inertia

*w* Width

*t* Thickness

*FEM* Finite Element Modeling

*CAD* Computer Aided Design

*λ* Empirical Natural Frequency Parameter

*σ* Empirical Natural Frequency Parameter

*y* Beam Displacement

*ρ* Density

*E* Young’s Modulus

*A* Cross-Sectional Area

*L* Length

*χ’* Dimensionless Length Parameter

*f* Natural Frequency

# Introduction

## Vibration of a Cantilever Beam

A cantilever beam is a defined as any long, constant cross-sectional area structure that is firmly fixed at one of its ends and free at the other. When a structure such as this is subjected to vibration, it behaves in very specific ways that have been well documented by engineers and scientists over the years. For example, if the thickness (t), width (w), and length (L) of the beam are known (And thus the moment of inertia of the beam, I), as well as its modulus of elasticity (E) and density (ρ), the natural frequencies of said structure can be found from the simple relationship listed below:

(1)

The i subscript listed on each side of the equation refer to the mode that said natural frequency is associated with. A mode is a state of oscillation in which each part of the beam oscillates with fixed phasing, in that, if one portion of the beam deflects a certain amount at a particular time and another beam deflects a different amount at that same time, there will exist a steady time over which said configuration repeats itself over and over. The number of a mode is associated with the number of nodal points that can be observed while it is taking place. A node point is a point at which there is no displacement of the beam relative to its neutral axis over the course of oscillation. If none of these nodal points exist, the mode number associated with the beam’s vibration is one. If one exists, the mode number is two, and so on and so forth.

For a simple cantilever beam, the location of the nodal points for a particular node can be found from the relationship listed below, where λ and σ are empirical values associated with finding the natural frequencies of constant area cantilever beams, and χ’ is the non-dimensional length said beam (x/L).

(2)

Based on these predictions, the actual natural frequencies and nodal points associated with a particular cantilever beam can be found through vibration testing. This is accomplished by setting the vibration frequency of the beam to those around the predicted natural frequencies, at which point specific values for the natural frequencies for each node can be found through the use of a strobe light. This is done by only altering the frequency of the strobe light until the beam appears to be stationary. The stationary wave that appears when this value is found is called a standing wave and represents the maximum deflection of the beam above and below its neutral axis for a given mode. With those results in mind, one can draw comparisons between the natural frequencies that were predicted by theory and those that were actually observed, as to assess the material properties associated with a given beam.

## Vibration of a Cantilever Wing

The definition of a cantilever wing is essentially the same as that for a cantilever beam, with the only difference between the two being the shape of the structure that is being secured. Though the definition of these two structures is similar, the way they behave when subjected to vibration is not. In the case of a cantilever beam, its width is small enough to discount any two-dimensional effects that may result from its vibration, effectively reducing it to a one-dimensional object that spans the length of the beam. For a wing, however, its area and width are large enough to allow for these two-dimensional effects to come into play, thus making it very difficult to develop standardized equations for its behavior when subjected to vibration.

With this in mind, there are two ways to go about finding the modal characteristics of a particular wing shape: Finite Element Modeling (FEM) Analysis and Experimentation. FEM Analysis is quick and simple, consisting of modeling the wing shape in a Computer Aided Design (CAD) program, setting up the fixtures associated with the wing (in this case, fixed at the root and free everywhere else), and then simulating vibrations in the vertical direction as to model the behavior of the wing at its various natural frequencies. This type of analysis works will for simplified shapes and fixtures (such as that of a thin, trapezoidal, cantilever beam), but becomes more time intensive and difficult as the complexity of the shape to be tested increases. However, the results that are obtained from this method are very precise and clear, making them a good reference source for physical testing.

Experimentation, on the other hand, allows for exact answers as to the natural frequencies associated with a given mode of vibration, as well as offers a general idea of the nodal lines associated with said modes. This is accomplished by coating the top of the wing with a fine, granular material (such as sand), and then adjusting the vibration frequency of the wing until the material begins to form specific shapes on the surface of the wing. The nodal points then are found based on the areas of the wing at which the material begins to congregate, since it is these areas where the least amount of displacement occurs in the vertical direction. Though it is clear where these nodal lines generally lie, they are by no means an exact measurement of where they occur in the wing itself. In order to have a more confident understanding of these nodal locations, it is absolutely necessary to compare them to those predicted by FEM analyses.

# Design of Test

## Cantilever Beam Test

The Cantilever Beam Test Apparatus consisted of a long, constant-area beam that was secured at its middle to a pneumatic, sinusoidal vibration apparatus. The frequency of the vibration apparatus was controlled through the use of a signal generator, while the frequency itself was measured through the use of a strobe light.

## Cantilever Wing Test

The Cantilever Wing Test Apparatus consisted of a trapezoidal, swept wing that was fixed at its root to a vibration apparatus. The frequency of the vibration apparatus was controlled by a function generator, while the nodal lines on the wing were found through the use of sand particles.

# Test Procedure

## Cantilever Beam Test Procedure

The procedure for conducting vibration testing of a cantilever beam is listed below:

1. The length, width, and thickness of the beam were measured through the use of a tape measure and caliper.
2. The area moment inertia of the beam was calculated by multiplying the width by the height cubed and then dividing the result by 12.
3. The theoretical natural frequencies of the beam were then found through the use of Equation (1) in Section (3).
4. The vibration frequency of the testing apparatus was then set to a value near that associated with the first natural frequency of the beam.
5. The lights of the testing area were then turned off and a strobe light was used to calculate the actual natural frequency associated with the first mode of the beam.
6. The location of the nodal points relative to the fixed end of the beam were then found through measurement
7. Steps 4-6 were then repeated for the second and third vibration mode of the beam.

## Cantilever Wing Test Procedure

The procedure for conducting vibration testing of a cantilever wing is listed below:

1. The dimensions of the wing were measured though the use of a caliper.
2. Sand was sprinkled over the top of the wing.
3. The frequency of the vibration apparatus was set to a value near that of the first mode of the wing.
4. The frequency of the vibration apparatus was adjusted until the sand particles located on the top of the wing began to form specific patterns on the top of the wing.
5. The frequency at which this occurred, as well as an image of the top of the wing, were recorded.
6. This same process was then repeated for three more modes of vibration.

# Test Results

## Cantilever Beam Test Results

Before any data was collected from experimentation, the geometry of the beam being tested was measured, its area moment of inertia was found, and its material properties were recorded, as listed in Table 6-1.

**Table 6-1. Beam Dimensions and Material Properties**

|  |  |
| --- | --- |
| **Beam Characteristics** | |
| Length (in) | 20.5625 |
| Width (in) | 1.02 |
| Thickness (in) | 0.055 |
| Moment of Inertia (in4) | 1.414x10-5 |
| Area (in2) | 0.0561 |
| Young’s Modulus (psi) | 10x106 |
| Density (lb-sec2/in4) | 25.55x10-6 |

Once these values were recorded, the theoretical natural frequencies of the beam were found through the use of Equation (1) in Section (3). These values were then compared to the natural frequencies obtained from the actual experiment. These values, along with the location of each nodal point, are listed in Table 6-1.

**Table 6-2. Theoretical and Actual Natural Frequencies/Nodal Points**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Mode** | **Frequencies (Hz)** | | **Nodal Point Locations (in)** | |
| **Theoretical** | **Actual** | **First** | **Second** |
| 1 | 3.95 | 4.16 | N/A | N/A |
| 2 | 23.35 | 26.05 | 16.25 |  |
| 3 | 66.5 | 72.95 | 17.75 | 10.0 |

## Cantilever Wing Test Results

Before any data was collected from vibration testing, the geometry of the wing was found and its material properties were recorded, as shown in Figure 6-1 and Table 6-3 respectively.



**Figure 6-1. PRO-E Model of Cantilever Wing**

**Table 6-3. Wing Material Properties**

|  |  |
| --- | --- |
| **Wing Characteristics** | |
| Young’s Modulus (psi) | 10x106 |
| Density (lb-sec2/in4) | 25.55x10-6 |

After this, the wing was then vibrated at frequencies close to those associated with its expected natural frequencies. For each natural frequency that was obtained, and image of the wing was captured, as to document the locations of each mode’s nodal points. The results of this process are shown in Table 6-4.

**Table 6-4. Theoretical/Actual Natural Frequencies/Nodal Lines**

| **Mode** | **Frequencies (Hz)** | | **Nodal Line Images** |
| --- | --- | --- | --- |
| **Theoretical** | **Actual** |
| 1 | 7.56 | 8.0 |  |
| 2 | 34.4 | 35.0 |  |
| 3 | 50.7 | 55.0 |  |
| 4 | 87.0 | 87.0 |  |

# Discussion of Results

## Cantilever Beam Discussion of Results

With the results in the previous section in mind, the next series of calculations that were made related to finding the theoretical nodal points for each mode of vibration of the beam. This was accomplished by adjusting the non-dimensional length parameter found in Equation (2) in Section (3) from zero to one for each mode of vibration. From these plots, the theoretical node points for each case were found based on the point where each shape crossed its x-axis. The results of this process are shown in Figures 7-1 through 7-3 and summarized in Table 7-1.

**Figure 7-1. First Mode Shape, Cantilever Beam**

**Figure 7-2. Second Mode Shape, Cantilever Beam**

**Figure 7-3. Third Mode Shape, Cantilever Beam**

**Table 7-1. Theoretical Node Point Locations**

|  |  |  |
| --- | --- | --- |
| **Mode** | **Nodal Point Locations (in)** | |
| **First** | **Second** |
| 1 | N/A | N/A |
| 2 | 17.20 |  |
| 3 | 15.2 | 10.40 |

When comparing these nodal point locations to those observed over the course of testing, one notes that the theoretical predictions listed in Table 7-1 map very closely to those found in Table 6-2. The largest percent error between any of the corresponding values on these tables is the 16.8% between the two first node points for mode three. This suggests that the beam used for testing, as well as the empirical values from which Equation (2) was derived, fall relatively in line with each other. The main reasons for any particular gap between these two values likely relate to inconsistencies in the beam’s material properties as well as wear and tear that it has experienced over years of testing, along with the relative difficulty of measuring the nodal points whilst the vibration apparatus was still running.

Another comparison that corroborates this evidence is that between the theoretical and actual natural frequencies of the beam, as listed in Table 6-2. The largest percent difference between any of these two values is that between the mode three natural frequencies, which came in at a value of 9.7%. Allowing with reasons listed in the previous paragraph, the main reason for discrepancies between the actual and theoretical values likely relate to estimation in determining the frequency at which a standing wave was observed with the strobe light. If more time was allotted to determining the actual natural frequencies, more accurate results would have been obtained.

With these observations in mind, this particular part of the experiment was fairly successful. The natural frequencies of a cantilever beam were calculated and observed based on analytical and empirical methods, the results of which mirrored each other in a statistically significant manner.

## Cantilever Wing Discussion of Results

# Conclusion